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N<sup>o</sup>. XVIII.

*Observations on the Theory of Water Mills, &c. by*  
W. WARING.

Read June  
15, 1792.

**B**EING lately requested to make some calculations relative to mills; particularly Doct. Barker's construction, as improved by James Rumsey, I found more difficulty in the attempt than I at first expected. It appeared necessary to investigate new theorems for the purpose, as there are circumstances peculiar to this construction, which are not noticed, I believe, by any author; and the theory of mills, as hitherto published, is very imperfect, which I take to be the reason it has been of so little use to practical mechanics.

The first step, then, toward calculating the power of any water-mill (or wind-mill) or proportioning their parts and velocities to the greatest advantage, seems to be,

*The correction of an essential mistake adopted by writers  
on the Theory of Mills.*

This is attempted with all the deference due to eminent authors, whose ingenious labours have justly raised their reputation and advanced the sciences; but when any wrong principles are successively published by a series of such pens, they are the more implicitly received, and more particularly claim a public rectification; which must be pleasing, even to these candid writers themselves.

George Atwood, M. A. F. R. S. in his masterly treatise on the rectilinear motion and rotation of bodies, published so lately as 1784, continues this oversight, with its pernicious consequences, through his propositions and corollaries (page 275 to 284) although he knew the theory was suspected: for he observes (page 382) "Mr. Smea-

" ten

“ton in his paper on mechanic power (published in the  
 “Philosophical Transactions for the year 1776) allows,  
 “that the theory usually given will not correspond with  
 “matter of fact, when compared with the motion of ma-  
 “chines; and seems to attribute this disagreement, rather  
 “to deficiency in the theory, than to the obstacles which  
 “have prevented the application of it to the complicated  
 “motion of engines, &c. In order to satisfy himself con-  
 “cerning the reason of this disagreement he constructed a  
 “set of experiments, which, from the known abilities  
 “and Ingenuity of the author, certainly deserve great con-  
 “sideration and attention from every one who is inter-  
 “ested in these inquiries.” And notwithstanding the same  
 “learned author says, “The evidence upon which the  
 “theory rests is scarcely less than mathematical.” I am  
 sorry to find, in the present state of the sciences, one of  
 his abilities concluding (page 380) “It is not probable  
 that the theory of motion, however incontestible its prin-  
 ciples may be, can afford much assistance to the practical  
 mechanic,” although indeed his theory, compared with  
 the above cited experiments, might suggest such an infer-  
 ence. But to come to the point, I would just premise these

### *Definitions.*

If a stream of water imping against a wheel in motion, there are three different velocities to be considered, appertaining thereto, viz.

First, the absolute velocity of the water :

Second, the absolute velocity of the wheel :

Third, the relative velocity of the water to that of the wheel, *i. e.* the difference of the absolute velocities ; or the velocity with which the water overtakes or strikes the wheel.

Now the mistake consists in supposing the momentum, or force of the water against the wheel, to be in the *duplicate ratio of the relative velocity*: Whereas.

*Prop. I.*

The force of an invariable stream, impinging against a Mill-Wheel in motion is in the *simple direct proportion of the relative velocity*.

For, if the relative velocity of a fluid against a single plane be varied, either by the motion of the plane, or of the fluid from a given aperture, or both, then, the number of particles acting on the Plane in a given time, and likewise the momentum of each particle, being respectively as the relative velocity, the force on both these accounts, must be in the *duplicate ratio of the relative velocity*, agreeably to the common theory, with respect to this *single plane*; but, the number of these planes, or parts of the wheel acted on in a given time, will be as the velocity of the wheel, or *inversely as the relative velocity*; therefore, the moving force of the wheel must be in the simple direct ratio of the relative velocity. Q. E. D.

Or, the proposition is manifest from this consideration; that, while the stream is invariable, whatever be the velocity of the wheel, the same number of particles or quantity of the fluid, must strike it some where or other in a given time; consequently, the variation of force is *only* on account of the varied impingent velocity of the same body, occasioned by a change of motion in the wheel; that is, the momentum is as the relative velocity.

Now, this true principal substituted for the erroneous one in use, will bring the theory to agree remarkably with the notable experiments of the ingenious Smeaton, before mentioned, published in the Philosophical Transactions of the Royal society of London for the year 1751, Vol. 51, for which the honorary annual medal was adjudged

judged by the society, and presented to the author by their president. An instance or two of the Importance of this correction may be adduced as follow.

*Prop. II.*

The velocity of a wheel, moved by the impact of a stream, must be half the velocity of the fluid, to produce the greatest possible effect.

For, let  $\left\{ \begin{array}{l} V = \text{the velocity, } M = \text{the momentum of the fluid} \\ v = \text{the velocity, } P = \text{the power of the wheel.} \end{array} \right.$

Then,  $V - v =$  their relative velocity, by definition 3d. and, as  $V : V - v :: M : \frac{M}{V} \times \overline{V - v} = P$  (*Prop. I.*) which  $\times v = P v = \frac{M}{V} \times v \overline{v - v^2} =$  a maximum; hence  $v \overline{v - v^2} =$  a maximum, and its fluxion, ( $v$  being the variable quantity)  $= V v - 2v v' = 0$ ; therefore  $v = \frac{1}{2} V$ , that is, the velocity of the wheel = half that of the fluid, at the place of impact, when the effect is a maximum. Q. E. D.

The usual theory gives  $v = \frac{1}{3} V$ ; where the error is not less than one third of the true velocity of the wheel!

This proposition is applicable to underhot wheels, and corresponds with the accurate experiments before cited, as appears from the Author's conclusion, (*Philosophical Transactions for 1776 page 457*) viz. "The velocity of the wheel, which, according to M. Parents determination, adopted by Desaguliers and Maclaurin, ought to be no more than one third of that of the water, varies at the maximum in the experiments of Table I. between one third and one half; but in all the cases there related, in which the most work is performed in proportion to the water expended and which approach the nearest to the circumstances of great works when properly executed, the maximum lies much nearer one half than one third, *one half seeming to be the true maximum*, if nothing were lost by the resistance of the air, the scattering of the water carried up by the wheel, &c." Thus

he fully shews the common theory to have been very defective; but, I believe, none have since pointed out wherein the deficiency lay, nor how to correct it; and now we see the agreement of the true theory with the result of his experiments.

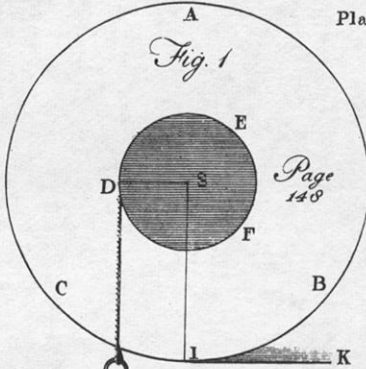
I might proceed with this correction through several propositions, &c. and shew their coincidence with those experiments; but must leave that, at present, for such as have more leisure; my view being only to shew where this perplexing difficulty crept in, in order that those who may have occasion to use the theory in future, or instruct young men in the principles of mechanics, may make any use of these hints they please: I will, however, just add one problem, as I have it by me; though it may not be the most suitable I could have chosen.

*Prop. III. Fig. 1. Plate 4.*

*Given*, the momentum (M) and velocity (V) of the fluid at I, the place of impact; the radius (R=IS) of the wheel ABC; the radius (r=DS) of the small wheel DEF on the same axle or shaft; the weight (W) or resistance to be overcome at D, and the Friction (F) or force necessary to move the wheel without the weight; *required* the velocity (x) of the wheel, &c.

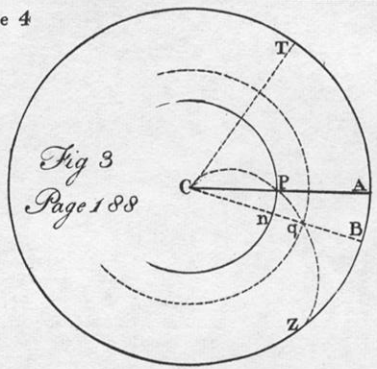
Here we have  $V : V - x :: M : M \times \frac{V-x}{V}$  = the acting force at I in the direction KI, as before. (prop. 2.) now,  $R : r :: W : \frac{rW}{R}$  = the power at I necessary to counterpoise the weight W; hence,  $\frac{rW}{R} + F$  = the whole resistance opposed to the action of the fluid at I; which deducted from the moving force, leaves  $M \times \frac{V-x}{V} - \frac{rW}{R} - F$ , = the accelerating force of the machine; which, when the motion becomes uniform, will be evanescent or = 0; therefore,  $M \times \frac{V-x}{V} = \frac{rW}{R} + F$ , which gives  $x = V \times \frac{1}{1 + \frac{rW + F}{MR}}$  = the true velocity required; or, if we reject the friction, then  $x = V \times \frac{1}{1 + \frac{rW}{MR}}$  is the the-

orem



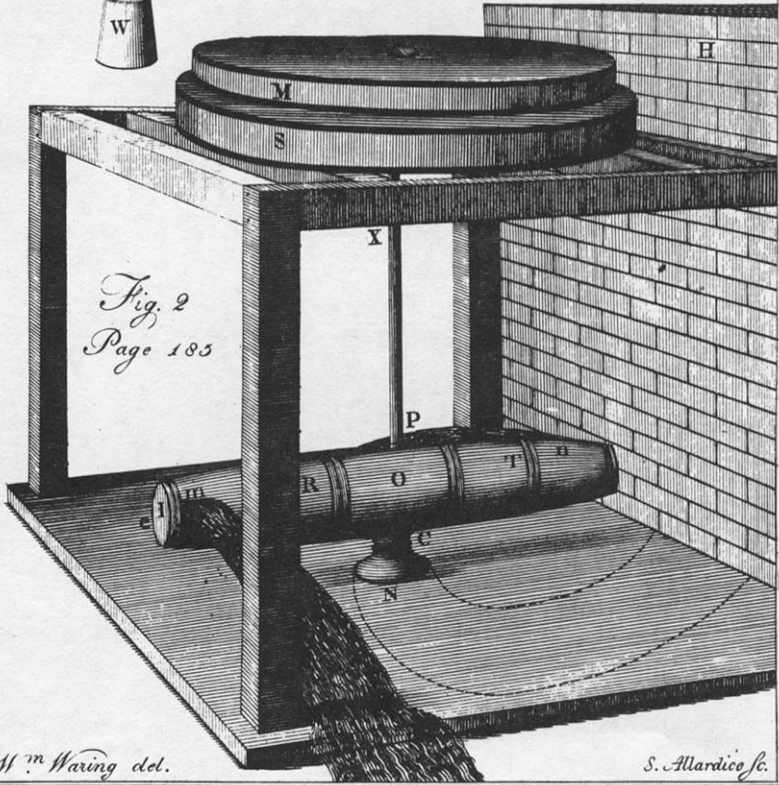
*Fig. 1*

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*Fig. 3*

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*Fig. 2*

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orem for the velocity of the wheel. This, by the common theory would be  $x = V \times \sqrt{\frac{rW}{MR}}$ , which is too little by  $V \sqrt{\frac{rW}{MR}} - V \frac{rW}{MR}$ : No wonder why we have hitherto derived so little advantage from the theory.

Corol. 1. If the weight (W) or resistance be required, such as just to admit of that velocity which would produce the greatest effect; then, by substituting  $\frac{1}{2}V$  for its equivalent  $x$  (by prop. II.) we have  $\frac{1}{2}V = V \times \sqrt{\frac{rW}{MR} - \frac{r}{M}}$ ; hence  $W = \frac{\frac{1}{2}M - F}{\frac{r}{MR}} \times R$ ; or, if  $F=0$ ,  $W = \frac{MR}{2r}$ ; but theorists make this  $\frac{MR}{9r}$ , where the error is  $\frac{MR}{18r}$ .

Corol. 2. We have also  $r = \frac{\frac{1}{2}M - F}{W} \times R$ ; or, rejecting friction,  $r = \frac{MR}{2W}$ , when the greatest effect is produced, instead of  $r = \frac{4MR}{9W}$ , as has been supposed: this is an important theorem in the construction of Mills.

WM. WARING

*Philadelphia, 7th, 9th mo. 1790.*

*Astronomical*